

Rep Tiles

Lesson Topic _____ **Grades** _____

Creating similar shapes using a single shape 4–5

Lesson Length _____

50–75 minutes

NCTM Standards Addressed _____

- Identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes.
- Investigate, describe, and reason about the results of subdividing, combining, and transforming shapes.
- Explore congruence and similarity.
- Make and test conjectures about geometric properties and relationships and develop logical arguments to justify conclusions.
- Predict and describe the results of sliding, flipping, and turning two-dimensional shapes.
- Build and draw geometric objects.
- Use geometric models to solve problems in other areas of mathematics, such as number and measurement.
- Understand such attributes as length, area, weight, volume, and size of angle and select the appropriate type of unit for measuring each attribute.
- Explore what happens to measurements of a two-dimensional shape such as its perimeter and area when the shape is changed in some way.

Sample State Standards Addressed _____

- Construct two- and three-dimensional shapes and figures using manipulatives, geoboards, and computer software.
- Describe the relationship between the perimeter and area of triangles, quadrilaterals, and circles.
- Identify properties of geometric figures (e.g., parallel, perpendicular, similar, congruent, symmetrical).

Student Objectives _____

Students will:

- develop a deeper understanding of similarity
- review names of geometric figures

- make predictions about what happens to the perimeter and area of a shape when it is used to create a similar figure.

Grouping for Instruction

- Whole class for launch and closure
- Small groups of four to six for the investigation

Overview of Lesson

Students use pattern blocks of one shape at a time to try to create a similar shape. They then compare the perimeter and area of the new figure with the perimeter and area of the original shape.

Background Information

Students should be familiar with basic shapes and understand that the name of the shape remains the same even if the orientation changes. Students should be familiar with the concept of similarity. Students should know how to find the perimeter and area of basic shapes. Familiarity with tilings of the plane would be beneficial. Students need some facility with a ruler.

Materials and Equipment

- Pattern blocks and rulers for each team
- Overhead projector
- Pattern blocks for the overhead projector

Procedure

A. Motivation and introduction

1. Use an overhead set of pattern blocks to show that four squares can be combined to create a new square that is similar to the original square. “Because the squares can be combined to create a similar square, we say the square is a ‘rep tile’ (for repeating tile). In fact, it is a rep-4 tile because it requires four squares to make a similar square.”
2. Remind students that for two shapes to be similar, corresponding angles must be congruent (the same measure) and corresponding sides must be proportional. Ask: “Is this true for the original square and the larger square just formed? Explain.”
3. Use six overhead squares to create a rectangle. Ask: “Is the larger shape similar to the original square? Why or why not?” “Is this new figure a rep tile?”
4. Ask: “Do you think other pattern blocks are rep tiles? How could you test your conjecture?”
5. Suggest that the students work on this problem in teams.

B. Development (including discussion points and feedback)

1. Place the students in heterogeneous cooperative groups.
2. Assign each student a task (leader, recorder, reporter, etc.).
3. Distribute the worksheet “Rep Tiles” (page 131).
4. Ask the groups to complete the “Rep Tile” investigation.
5. Circulate among the teams, guiding the students to complete the project, and observing how students work and understand the concepts.
6. Ask questions that will help students understand that they may have to rotate or flip the pattern blocks to make a similar shape.
7. Make sure each team compares the perimeters of the similar shapes.
8. Make sure each team compares the areas of the similar shapes.
9. Ask each team to report on one part of the investigation.
10. Ask questions to assess whether the students understand the concepts of perimeter, area, similarity, and the patterns discovered. Examples are:
 - “If these shapes are similar, what must be true about the corresponding angles?”
 - “What does it mean for corresponding sides to be proportional?”
 - “How did you find the perimeter? The area?”
 - “Justify your pattern. How do you know it always holds?”
11. Encourage the students to question whether each team has in fact found a rep tile, and ask for the patterns found.
12. Have students develop equations for each pattern. (For a rep-4 tile, the perimeter of the larger shape is twice the perimeter of the original shape.)

C. Summary and closure

1. Ask students to write three mathematics concepts that they did today (area, perimeter, similarity, rep tiles, etc.).
2. Have some students share what they wrote. Explore students’ understanding. (Did they learn what you wanted them to learn?)
3. Ask students why they think these concepts are important.

D. Assignment

Give examples of polygons that are not convex (as are all of the pattern blocks). Ask students to find a non-convex polygon that is a rep tile for homework. Students should show a picture of their non-convex rep tile and how its similar shape was formed.

Assessment

- Observe the students during the investigation. Take notes on individual students' ability to approach this task and their understanding of the concepts.
- Grade the group project, giving each team a group grade.
- Grade the homework, if you give them a rubric first.

Worksheet:

Rep Tiles

A Geometry Project Using Pattern Blocks

A **rep tile** is a polygon that will tile the plane. For example, a square is a rep tile. We say the square is a rep-4 tile because it requires 4 congruent squares to form a similar figure—in this case a larger square.

Recall:

- Two polygons are **congruent** if the corresponding sides are the same length and the measures of corresponding interior angles are the same.
- Two polygons are **similar** if the corresponding sides are proportional—the ratios of corresponding sides equal a constant k —and corresponding interior angles have the same measure.

1. Trace a square pattern block in the space to the right.
2. Measure the sides to find the dimensions of the square.
3. What is the perimeter of the square?
4. What is the area of the square?
5. Fill in the perimeter and area of the square (columns 2 and 4) on the first row of the table on the next page.
6. Use four squares to create a larger square and trace the larger square in the area to the right.
7. Find the perimeter and area of the larger square.
8. Fill in the perimeter and areas of the rep tile (columns 3 and 5) on the first row of the table on the next page.
9. Explore to determine which of the other pattern blocks are rep-4 tiles. Complete the table on the next page by recording the name of each pattern block that is a rep-4 tile and the perimeters and areas of the original polygon and similar polygon.

Pattern Block Name	Perimeter of Pattern Block	Perimeter of Rep Tile	Area of Pattern Block	Area of Rep Tile
Square				

10. Refer to the table above. Look for a pattern between the perimeter of the original pattern block and the perimeter of the rep-4 tile formed using this pattern block. Describe the pattern using your own words.

11. Describe the pattern using an equation.

12. Refer to the table above. Look for a pattern between the area of the original pattern block and the area of the rep-4 tile formed using this pattern block. Describe the pattern using your own words.

13. Describe the pattern using an equation.

14. Suppose the perimeter of a polygon is 12 centimeters and its area is 10 square centimeters. If four of these polygons form a similar polygon, what is the perimeter and area of the new, larger polygon?

Perimeter:

Area:

15. Suppose it requires 9 copies of a polygon to form a similar polygon. What do you think would have to be true about the relationship between the **perimeter** of the original polygon and the perimeter of the larger, similar polygon made using 9 of the original polygon?
16. Suppose it requires 9 copies of a polygon to form a similar polygon. What do you think would have to be true about the relationship between the **area** of the original polygon and the area of the larger, similar polygon made using 9 of the original polygon?
17. Check your conjectures by using 9 squares to create a square and then measuring to find the perimeter and area.
- Were you correct?
 - Do you want to revise your conjectures?
 - If you want to change your conjecture, what new conjecture would you make?
18. Notice that the perimeter of a similar shape requiring 4 polygons has a perimeter 2 times the perimeter of the original polygon and an area 4 times the area of the original polygon. Also, the perimeter of a similar shape requiring 9 polygons has a perimeter 3 times the perimeter of the original polygon and an area 9 times the area of the original polygon. Suppose it requires 16 polygons to make a similar shape. What must be true about the perimeter and area of the similar shape? Explain how you know this.
19. There is a triangle that is a rep-2 tile. What is it? Use what you have learned to sketch the triangle and the similar triangle formed using two of the original triangles. Find the area and perimeter of the two triangles.

20. Can you find a polygon that is a rep-3 tile? Show it and the similar polygon formed in the space below. Find the perimeter and area of each figure.

21. Can you find a rep-5 tile? Show it and the similar polygon constructed in the space below.