Tiling the Plane

Lesson Topic

Tiling the plane with pattern blocks

Grade

5

Lesson Length

50 minutes

NCTM Standards Addressed

- Recognize, name, build, draw, compare, and sort two- and three-dimensional shapes.
- Investigate and predict the results of putting together and taking apart two- and three-dimensional shapes.
- Predict and describe the results of sliding, flipping, and turning two-dimensional shapes.
- Select and apply appropriate standard units and tools to measure length, area, volume, weight, time, temperature, and the size of angles.

Sample State Standards Addressed

- Give formal definitions of geometric figures. Construct two- and three-dimensional shapes and figures using manipulatives, geoboards, and computer software.
- Create an original tessellation.
- Represent and use the concepts of line, point, and plane.
- Analyze simple transformations of geometric figures and rotations of line segments.

Student Objectives

Students will:

- develop a deeper understanding of tessellations (tiling the plane)
- review names of geometric figures.

Grouping for Instruction

- Whole class for launch and closure
- Small groups of four to five for the investigation

Overview of Lesson

In this lesson students will use pattern blocks and triangular grid paper to review shape names, be introduced to the concept of a tiling of the plane, and determine which pattern blocks will tile the plane. This information will be used to determine the measures of the interior angles of the regular pattern blocks that will tile the plane.

Background Information

Students should have had an opportunity to play with pattern blocks prior to starting this lesson. Students should be familiar with basic shapes and understand that the name of a shape remains the same even if its orientation changes. Students may or may not be familiar with the word "tessellation" or the concept of tiling.

Materials and Equipment

- A set of overhead pattern blocks
- A classroom set of pattern blocks
- Several sheets of triangular grid paper per student
- Triangular grid paper for the overhead projector
- Overhead projector
- Examples of M.C. Escher's art that use tessellations.

Procedure

A. Motivation and introduction

1. Ask students if they know what a tessellation is. Same for a tiling. Ask where they might have seen a tessellation or tiling.
2. Use an overhead set of pattern blocks to show that the equilateral triangle will cover the overhead screen with no overlapping and no spaces between the triangles.
3. State that (or ask if) this is an example of a tessellation—a tiling of the plane.
4. Show an overhead transparency in which regular pentagons only partially cover the screen (there are gaps between the pentagons). Ask: "Is this a tiling of the plane? Why or why not?"
5. Show an attempt at a tiling in which the polygons overlap. Ask: "Is this a tiling of the plane? Why or why not?"
6. Show the students a copy of the triangular grid paper on the overhead. Show how the triangle and the trapezoid can be placed on the paper and have the sides "line up" with sides of the triangles on the grid paper.
7. State: "The triangle tessellates or tiles the plane. Do you think other pattern blocks will tile the plane? How could you test your conjecture?"
8. Suggest that the students work on this problem in teams.

B. Development (including discussion points and feedback)

1. Place the students in heterogeneous cooperative groups of about four students.
2. Assign each student a task (leader, recorder, reporter, materials person, etc.).
3. Have the materials persons for each group come up and get for their group the pattern blocks, the triangular grid paper, and the investigation worksheet: "Pattern Blocks, Triangle Grid Paper, and Tilings" (page 153).
4. Ask the teams to complete the investigation.
5. Circulate among the teams, guiding the students to complete the project, and observing the groups' approaches and individual students' participation and understanding of the process and concepts.
6. Ask questions that will help students understand that they may have to rotate or flip the pattern blocks to make them tile the plane.
7. Ask each team to report on one pattern block that tessellates the plane, and show a sample of a tessellation.
8. Ask questions to assess whether the students understand what a tessellation is.
    - "Do you agree that this is a tessellation?" (EPR: thumbs up to show agreement, thumbs down to show disagreement.)
    - "Why is this a tessellation?"
    - "If the pieces overlap, is it still a tessellation?"
9. Encourage the students to question whether each team has in fact found a tiling.
10. Show examples of some of the tilings of M. C. Escher on the overhead projector, if possible. Ask the students if they can recognize which shape Escher used with each tiling and how he used that shape.
11. Ask the students if Escher used rotations or flips in creating the tiling.
12. Tell students that they should try this. Perhaps one day they might be an artist like M.C. Escher and design tessellations.

C. Summary and closure

1. Ask groups to collaborate to write:
    - One sentence on what we learned (did) today.
    - Two places where tessellations (tilings) may be seen in nature or used in a real-life situation.
2. Have groups share their responses and build on their responses to summarize the following:
    - A tessellation or tiling of a plane occurs when a figure is able to cover a plane completely with no gaps and no overlaps.
    - Tessellations are found in nature and in real life in a honeycomb, patterns in floors, walls, buildings, works of art, etc.

D. Assignment

Give examples of polygons that are not convex (as are all of the pattern blocks). Ask the students to create a non-convex polygon that will tile the plane for homework. Students should be encouraged to create a picture using their tiling using M. C. Escher as an inspiration. Have the class develop a rubric for assessing the assignment.

Assessment

- Observe the students during the investigation using a checklist to note what students know and any areas that need to be addressed again.
- Grade the group project, giving each team a group grade.
- Have students assess the tilings they created for homework using a rubric the class developed.
- Give students practice in assessing their own work using the rubric.

Worksheet:

Pattern Blocks, Triangle Grid Paper, and Tessellations

1. Refer to the triangle grid paper provided. This paper shows one way that the green equilateral triangle pattern block could be used to **tessellate** (**tile) the plane** (completely cover an infinite plane with one or more figures). Using the triangle grid paper and the pattern blocks, show how you could tessellate the plane with the blue **rhombus** (a 4-sided figure with all sides the same length). Use different colors to show the tessellation.

2. Using the triangle grid paper and the pattern blocks, show how you could tessellate the plane with the **regular hexagon** (a 6-sided figure with all sides the same length and all interior angles the same measure) pattern block. Use different colors to show the tessellation.

3. Using the triangle grid paper and the pattern blocks, show how you could tessellate the plane with the **trapezoid** (a 4-sided figure in which two opposite sides are parallel and the other pair of opposite sides are not parallel) pattern block. Use different colors to show the tessellation.

4. Using the triangle grid paper and the pattern blocks, show how you could tessellate the plane using two different pattern blocks. Show your tessellation using different colors.

5. Can you tessellate the plane with a different pattern block? Convince me you are correct.

6. How many equilateral triangles meet at a point when they are used to tile the plane? Since the total of all the angles around the point is 360 degrees, what must be the measure of one angle of the equilateral triangle?

7. What is the measure of each angle of the regular hexagon? How do you know?